

A short probability problem

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Question

Tom and Mary throw a fair die alternatively with Tom throwing first.

If the first who gets a '6' wins, what is the probability that Tom will win?

Solution 1 Using infinite Geometric series

$$\begin{aligned} & P(\text{Tom wins}) \\ &= P(\text{Tom wins in 1}^{\text{st}} \text{ throw}) + P(\text{Tom wins in 3}^{\text{rd}} \text{ throw}) + P(\text{Tom wins in 5}^{\text{th}} \text{ throw}) \\ &+ \dots \\ &= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \dots \\ &= \frac{1}{6} + \frac{1}{6} \times \left(\frac{5}{6}\right)^2 + \frac{1}{6} \times \left(\frac{5}{6}\right)^4 + \dots \\ &= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \end{aligned}$$

Solution 2

$$P(\text{Tom wins}) + P(\text{Mary wins}) = 1 \quad \dots (1)$$

Now, if Tom loses in the first throw, then the status of Tom and Mary interchange.

$$\begin{aligned} \therefore P(\text{Mary wins}) &= P(\text{Tom loses in 1st throw and Tom wins}) \\ &= P(\text{Tom loses in 1st throw}) \times P(\text{Tom wins}) \\ &= \frac{5}{6} \times P(\text{Tom wins}) \quad \dots (2) \end{aligned}$$

$$\text{Sub. (2) in (1), } P(\text{Tom wins}) = \frac{6}{11}, \quad P(\text{Mary wins}) = \frac{5}{11}$$

Solution 3

If both Tom and Mary lose, that is, not '6' in the first 2 throws, then they just like to restart the game.

Let $a = P(\text{Tom wins})$, then

$$a = P(\text{Tom wins in the first throw}) + P(\text{not '6' in the first 2 throws}) \times a$$

$$a = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times a \quad \Rightarrow \quad a - \left(\frac{5}{6}\right)^2 \times a = \frac{1}{6} \quad \Rightarrow \quad \frac{11}{36} \times a = \frac{1}{6}$$

$$\text{Solving, } \therefore a = \frac{6}{11}$$